RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, MARCH 2021

SECOND YEAR [ARREAR]

Date : 30/03/2021 **MATHEMATICS FOR INDUSTRIAL CHEMISTRY [General]**

Time : 11 am – 1 pm **Paper : III** Full Marks : 50

Group – A

Answer any three questions from Question nos. 1 to 5 : [3×5]

1. Show that the area of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is
$$\frac{\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$$
2. Reduce the equation $x^2-3xy+y^2+10x-10y+20=0$, to canonical form. What is the nature of the curve? [4+1]
3. If the straight line $r\cos(\theta-\alpha)=p$ touches the parabola $\frac{l}{r}=1+\cos\theta$, show that $p=\frac{l}{2}\sec\alpha$.
4. Find the shortest distance between the lines
$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-3}{3}=\frac{y-3}{4}=\frac{z-4}{5}$.
5. Find the equation of the plane which passes through the point (2,1,4) and perpendicular to each of the planes
$9x-7y+6z+48=0$ and $x+y-z=0$.

Group – B

Answer any four questions from Question nos. 6 to 11 : [4×5]

6. a) Find the order and degree of $\left(\frac{d^2y}{dx^2}\right)^2+\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$. [2]

b) Eliminate the arbitrary constants A and B from the relation $y=\frac{A}{x}+B$ and obtain a differential equation of second order [3]

7. Solve : $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$
8. Find the general and singular solutions of the differential equation
$y=px+ap(1-p)$, where $p=\frac{dy}{dx}$.
9. Solve : $\frac{dy}{dx}=\frac{x+y+1}{2x+2y+1}$
10. Solve by method of variation of parameters
$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}.$$

11. Show that the orthogonal trajectories of $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$, λ being arbitrary, is $x^2+y^2+c=2a^2 \log x$.

Group – C

Answer <u>any three</u> questions from <u>Question nos. 12 to 16</u> : [3×5]

12. a) Examine if the set S is a subspace of $\mathbb{R}^3$ or not, where
$S=\{(x,y,z)\in\mathbb{R}^3 : x=z=0\}$ [3]

b) Determine k so that the set S is linearly dependent in $\mathbb{R}^3$, where $S=\{(1,2,1),(k,3,1),(2,k,0)\}$. [2]

13. Show that $\{(1,2,3),(2,3,1),(3,1,2)\}$ forms a basis in $\mathbb{R}^3$.

14. Find a basis for the vector space $\mathbb{R}^3$, that contains the vectors $\{(1,0,1),(1,1,1)\}$.

15. Consider the map $T:\mathbb{R}^3\to\mathbb{R}^3$ be a linear transformation defined by $T(x_1,x_2,x_3)=(2a_2+a_3,-a_1+4a_3,5a_2)$. Find $[T]_\beta$, where $\beta=\{(1,0,0),(0,1,0),(0,0,1)\}$.

16. Consider the map $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x_1,x_2,x_3)=(x_1+2x_2+x_3,x_1+x_2,0)$

a) Show that T is a linear transformation.

b) Is T is one-one? Justify. [3+2]

———— × ————